中原大學 96 學年度碩士班入學考試

96/03/25 14:00~15:30 電子工程學系通訊組

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目： 工程數學(線性代數 50%、機率 50%)　　（共 1 頁第 1 頁）

■可使用計算機，惟僅限不具可程式及多重記憶者　□不可使用計算機

1. (20%) Is the given set (taken with the usual addition and scalar multiplication) a vector space or not? (Give a reason.) If your answer is yes, determine the dimension and find a basis.
   a. All vectors in $\boldsymbol{R}^3$ satisfying $v_1 - 3v_2 + 2v_3 = 0$.
   b. All polynomials in x, of degree not exceeding 3.
   c. All skew-symmetric 2×2 matrices.
   d. All 3×3 matrices with main diagonal 1, 1, 1.
2. (15%) Use Gram-Schmidt Orthogonalization process to find a orthonormal basis for the subspace $\mathbf{W}$ = span {(1,0,-1,0), (1,-1,0,0), (3,1,0,0)} in $\boldsymbol{R}^4$.
3. (15%) A zero-mean Normal random vector $\mathbf{X} = (X_1, X_2)^{\mathrm{T}}$ has covariance matrix $\mathbf{K}$ given by $\mathbf{K}=\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$. Find a transformation $\mathbf{Y} = \mathbf{DX}$ such that $\mathbf{Y} = (Y_1, Y_2)^{\mathrm{T}}$ is a Normal random vector with uncorrelated (and therefore independent) components of unity variance.

   hint: $f_{\mathbf{X}}(\mathbf{x}) = \dfrac{1}{(2\pi)^{n/2}[\det(\mathbf{K})]^{1/2}} \exp(-\dfrac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \mathbf{K}(\mathbf{x}-\boldsymbol{\mu}))$, where $\boldsymbol{\mu}$ is the mean vector of $\mathbf{X}$ and n is the number of elements of $\mathbf{X}$, herein $\boldsymbol{\mu} = [0, 0]^{\mathrm{T}}$ and $n = 2$.
4. (20%) Consider a random variable X with possible outcomes: 0, 1, 2, $\cdots$. Suppose that the probability $P(X = j) = (1-c)c^j$, $j = 0,1,2,\cdots$.
   a. For what value of c is the above model meaningful?
   b. Show that for any two positive integers s and t,
      $P(X > s+t \mid X > s) = P(X \ge t)$.
5. (10%) Let X be a continuous random variable having cumulative distribution function F. Define the random variable Y by $Y = F(X)$. Show that Y is uniformly distributed over [0,1].
6. (20%) Let $X_1, X_2, ..., X_n$ be i.i.d. (independent and identically distributed) random variables with $E[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2$. Let $\hat{\mu} = \dfrac{1}{n}\sum_{i=1}^{n} X_i$. What is the value of a in $\theta = a\hat{\mu}$ will generate the minimum-mean-square-error estimator of μ? (i.e., find the optimum value of $a = a_O$ to minimize $\mathrm{E}[(\theta-\mu)^2]$) When $n \to \infty$, $a_O \to ?$