

# 中原大學 95 學年度碩士班入學考試

3 月 18 日 14:00~15:30 電子工程系通訊組

誠實是我們珍視的美德，  
我們喜愛「拒絕作弊，堅守正直」的你！

科目：工程數學

(共 2 頁第 1 頁)

■ 可使用計算機，惟僅限不具可程式及多重記憶者

不可使用計算機

1. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Calculate  $\det A$ . Use the value of  $\det A$  to answer the following questions:

- Is the rank of  $A$  equal to 3?
- Does the system of equations  $AX=B$  have a unique solution for every  $B$ ?
- Does  $A^{-1}$  exist?
- Does the system of equations  $AX=0$  have nontrivial solutions?

16%

2. The equation of a line in the plane has the form  $ax+by+c=0$ . Two distinct points determine a line. We wish to find the equation of the line determined by the points  $(1,2)$  and  $(5,7)$ . Since  $(1,2)$  lies on the line, we know  $a+2b+c=0$ . Since  $(5,7)$  lies on the line, we know  $5a+7b+c=0$  also. Find  $a$ ,  $b$ , and  $c$  not all zero so that all following equation holds

14%

$$ax + by + c = 0$$

$$a + 2b + c = 0$$

$$5a + 7b + c = 0$$

3. Let  $v=(2,5,-6,4)$ . Does  $v$  belong to the space spanned by  $\{u_1, u_2, u_3\}$ , where  $u_1=(1,2,-1,1)$ ,  $u_2=(0,1,-4,2)$ , and  $u_3=(1,1,3,-1)$ ? Prove it.

20%

4. Consider the probability density

$$f(x) = ae^{-b|x|}$$

where  $X$  is a random variable whose allowable values range from  $x=-\infty$  to  $x=+\infty$ . Find

- The cumulative distribution function  $F(x)$ ,
- The relationship between  $a$  and  $b$ ,
- The probability that the outcome  $X$  lies between 1 and 2.

18%

5. Consider a random variable  $X$  with having known probability density as following

$x$	-3	-1	0	1	2	3	5	8
$f(x)$	0.1	0.2	0.15	0.2	0.1	0.15	0.05	0.05

Find the following probabilities:

- $X$  is negative number
- $X$  is even number
- $1 \leq X \leq 8$
- $p(X=-3|X \leq 0)$
- $p(X \geq 3|X > 0)$

20%

6. Consider the random process

$$V(t) = \cos(\omega t + \theta)$$

where  $\theta$  is a random variable with a probability density

$$f(\theta) = \begin{cases} \frac{1}{2p}, & -p \leq \theta \leq p \\ 0, & \text{elsewhere} \end{cases}$$

- Show that the first and second moments of  $V(t)$  are independent of time?
- If the random variable  $\theta$  is replaced by a fixed angle  $\theta=30^\circ$ , will the ensemble mean of  $V(t)$  be time independent?

12%