

# 中原大學 96 學年度碩士班入學考試

96/03/25 11:00~12:30 資訊工程學系

誠實是我們珍視的美德，  
我們喜愛「拒絕作弊，堅守正直」的你！

科目：計算機數學(含離散數學與線性代數)

(共 2 頁第 1 頁)

可使用計算機，惟僅限不具可程式及多重記憶者  不可使用計算機

1. [12%] Determine whether each of the following statements is **True or False**.
  - (a) There is a tree with degrees 3, 2, 2, 2, 1, 1, 1, 1, 1. \_\_\_\_\_
  - (b) In a group of *five* persons, at least *two* of them have the same number of friends in this group. \_\_\_\_\_
  - (c) Let  $2 \in S$ , and let  $x-y \in S$  if  $x, y \in S$ . Then  $S$  is the set of all even integers. \_\_\_\_\_
  - (d) If the composite function  $f \circ g$  is a bijection, then the function  $f$  must be one-to-one and the function  $g$  must be onto. \_\_\_\_\_
  - (e) If  $ac \equiv bc \pmod{m}$ , then  $a \equiv b \pmod{m}$ . \_\_\_\_\_
  - (f)  $\forall x \forall y P(x, y) \rightarrow Q(x, y)$ , where  $P(x, y)$  means " $x \geq y$ ,"  $Q(x, y)$  means " $x^2 \geq y^2$ ," and the universe of discourse is the set of all non-negative integers. \_\_\_\_\_
  
2. [24%] **Fill in the blanks** in the following statements.
  - (a) If a forest  $F$  consists of 9 trees and 69 vertices, then  $F$  has (1) edges.
  - (b) There can be (2) different equivalence relations on the set  $\{a, b, c, d\}$ .
  - (c) The number of bit strings of length *six* with no *five* consecutive 0s is (3).
  - (d) There are (4) ways to select two numbers from  $\{1, 2, 3, 4, 5, 6, 7\}$  so that the sum is even, e.g.,  $\{1, 1\}$  and  $\{2, 4\}$ .
  - (e)  $7^{360} \pmod{180} = \underline{(5)}$ .
  - (f) The *big-O* estimate for  $f(n) = (n! + 5^n)(n^n + \log(n!) + 5^n) = O(\underline{(6)})$  by using a simple function of the smallest order.
  - (g) There are (7) consecutive 0s at the end of the binary expansion of  $80!$ .
  - (h) Given the matrices  $A, B, C, D$  with sizes  $6 \times 10, 10 \times 5, 5 \times 2, 2 \times 3$  respectively, the most efficient way to compute the product  $ABCD$  is (8).

3. [7%] Consider the game rule: “Given 17 stones in a pile, two players take turns removing 1, 2, or 3 stones, and whoever takes the last stone loses.” Design a strategy to guarantee one player a win and show your answer step by step.

4. [7%] Draw the ordered rooted tree that corresponds to the arithmetic expression:  $* / 8 4 + * 7 5 - 6 3$ , which is written in prefix notation. Then, rewrite this expression in postfix notation and compute the value.

5. [12%]  $X(t+1)=AX(t)$ , with  $X(0)=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ,  $A=1/5\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ .

Determine  $X(10)$

6. [10%] Orthogonally diagonalize  $A$  and determine  $A^6$

$$\text{but } A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

7. [10%] Transform  $u_1, u_2, u_3$  to an orthonormal vectors  $v_1, v_2, v_3$  under the Euclidean inner product using the Gram-Schmidt process (beginning from  $u_1$ ).

$$u_1=(1,0, 0), u_2=(1,1, 1), u_3=(0,2, 1).$$

8. [8%] Determine a 2D transformation matrix ( $A$ ) that is a concatenation of reflection to  $y$ -axis, followed by a rotation of 30 degree. Also determine  $A^{-1}$ .

9. [10%] Determine the kernel space and the range of the linear transformation

$$T(X), \text{ but } T(X)=AX,$$

$$A = \begin{bmatrix} 1 & -1 & 2 & -2 \\ 0 & 2 & -4 & 0 \\ 0 & -2 & 4 & 0 \end{bmatrix}$$