中原大學 95 學年度碩士班入學考試

3 月 18 日 11:00~12:30 資訊工程系

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：計算機數學 **（共　頁第　頁）**

□可使用計算機，惟僅限不具可程式及多重記憶者　□不可使用計算機

1. **[42%]Fill in the blanks** in the following problems.

   A. If ***a*** and ***b*** are integers and ***m*** is a positive integer, then ***a is congruent to b modulo m*** if ***m*** divides ***a-b***. We use the notation $a \equiv b (\text{mod } m)$ to indicate that ***a*** is congruent to b modulo ***m***. Let **A**={1,2,3,4,5,6,7,8} and relations $R_2 = \{(x, y) \mid x \equiv y (\text{mod } 2), x, y \in A\}$, $R_3 = \{(x, y) \mid x \equiv y (\text{mod } 3), x, y \in A\}$, the **set of equivalence classes** *induced* by R_2 on **A**=___(1)___; the number of element of R_3 =___(2)___; $R_2 \cap R_3$ =___(3)___.

   B. Let ***L(x)*** and ***H(x)*** be the statements "*x* is large" and "*x* is a hummingbird", respectively. Assuming that the universe of discourse is the set of all birds. The statement "**Hummingbirds are all small**." can be expressed as a logical expression involving *predicates, quantifiers* and *logical connectives* as ___(4)___.

   C. If **A**={a,b,c,d} and **B**={1,2,3}, then (a) there are ___(5)___ functions from **A** to **B**; (b) there are___(6)___ ***injective*** functions; and there are___(7)___ ***surjective*** functions.

   D. Let **S**={3,7,11,15,19,…,95,99,103}. How many elements must we select from **S** to insure that there will be *at least two* whose sum is **110**? Answer=___(7)___.

   E. There are ___(8)___ *linear arrangements* of 9 letters in **CHUNGYUAN**.

   F. The *generating function* for the sequence $1^2, 2^2, 3^2, \ldots$ is___(9)___.

   G. The solution for the recurrence relation $a_n = a_{n-1} + (n-1)$, $n \geq 2$, $a_1 = 5$ is___(10)___.

   H. Let $a_1, a_2, a_3, \ldots$ be the integer sequence defined by recursively by (a) a_1=1; and (b) for all $n \in Z^+$, where $n \geq 2$, $a_n = 2a_{\lfloor \frac{n}{2} \rfloor}$, then a_8=___(11)___.

   I. Given a big-O estimate for $f(n) = n\log(n!) + n^2 + (\sin n)^4$ use a simple function of smallest order, *f(n)*= ___(12)___.

   J. Let **T=(V,E)** be a *complete* 3-ary tree with 34 *internal vertices*. Then **T** has___(13)___ vertices and ___(14)___ leaves.

2. **[8%]**Suppose that $f : Z^+ \to R$ with *f(1)=7*, and $f(n) = 4f(\frac{n}{3}) + 7$, for $n=3^k$, $k \in Z^+$. Solve for *f(n)* relative to the set $S = \{3^k \mid k \in N\}$.

3. **[5%]** The following sets of vectors are linearly dependent? **可不寫計算過程**

(a) u=(10, 11, 15, -4), v=(4,5,6, -1), x=(2,2,3, -1)

(b) u=(1,0, 100), v=(9,9,10) , w=(7,8,14), x=(7,8,9)

(c) u=(1,2,4), v=(4,5,6), w=(7,11,18)

(d) u=(10,11,15, 4), v=(4,5,6, 2), x=(2,2,3, 1)

(e) $u_1=3-x-x^2$, $u_2=3+2x^2$, $u_3=x+3x^2$

4. **[10%]** Transform u_1, u_2, u_3, u_4 to an orthonormal basis v_1, v_2, v_3, v_4 under the Euclidean inner product using the Gram-Schmidt process (beginning from u_1). u_1=(3, 0, 0, 0), u_2=(0, -2, 0, 0) , u_2=(0,-2, 2, 2), u_2=(0, 0, -1, 3).**可不寫計算過程**

5. **[15%]** For a linear transformation AX=W, (from R^4 to R^3,) $A=\begin{bmatrix} 2 & 4 & 0 & 1 \\ 1 & 2 & -1 & 2 \\ 0 & -1 & 2 & 3 \end{bmatrix}$

Write the kernel (a vector space in which the vectors are transformed to the zero vector.) and the range (a vector space that includes all transformed vectors but no other vectors.).

6. **[20%]** (a) Determine a P to orthogonally diagonalize the matrix B and (b) Solve the system of differential equations.

(a)

$$B=\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

(b)

$$\begin{aligned} y_1' &= y_1 - y_2 + y_3 \\ y_2' &= -y_1 + y_2 - y_3 \\ y_3' &= y_1 - y_2 + y_3 \end{aligned}$$

initial values for y_1, y_2, y_3 are 0, 1, -1