

中原大學 94 學年度碩士班入學考試

3 月 20 日 14:00~15:30 資訊工程系

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：資料結構與演算法

(共 3 頁第 1 頁)

可使用計算機，惟僅限不具可程式及多重記憶者

不可使用計算機

註：請按順序作答，並標明答案

1. Given a called-by-value subprogram as:

Procedure *subp1(a, b)*

begin;

$a := 2 * a + b;$

end.

And another called-by-address subprogram is:

Procedure *subp2(a, b)*

begin;

$b := a - b;$

end.

If $a = 3$ and $b = 4$ are chosen initially. After both subprograms are called once, what's the value of $a + b$? (5%)

- Let $v = n^{\log n}$, $w = (\log n)^{100}$, $x = \log n^n$, $y = n + \log n$ and $z = \log \log n$. Arrange the five functions by their orders of growth rate in increasing order. (5%)
- Given A as an N by N array ($N \geq 2$) with $A[i, j] = 0$ if $|i - j| > 2$. Then, determine the maximal number of nonzero elements in A . (5%)
- Let x, y, z and w represent binary operators, and A, B, C, D and E represent data items (variables or constants). The precedence of operators are “ x over y ”, “ y over z ” and “ z over w ,” and the transitivity holds. Determine the prefix of $(AzBxC)y(DwE)$. (5%)
- For a general tree of degree 5, if the node number is 400, what's the minimum depth of the tree? (5%)
- Consider inserting the keys 10, 33, 5, 15, 28, 44, 26 into a hash table of length $m = 11$ using the division method (i.e., the hash function is: $h(k) = k \bmod m$). Illustrate the result of inserting these keys using linear probing. Also determine the number of collisions. (10%)

7. Given the following QUICKSORT algorithm to sort the array A with n numbers (i.e., $p = 1, r = n$):

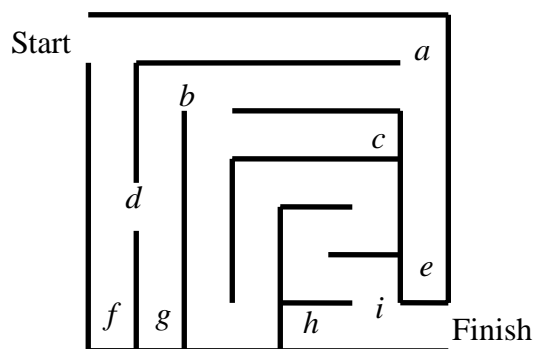
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QUICKSORT( $A, p, r$ )
1  if  $p < r$ 
2    then  $q \leftarrow$  PARTITION( $A, p, r$ )
3         QUICKSORT( $A, p, q - 1$ )
4         QUICKSORT( $A, q + 1, r$ )

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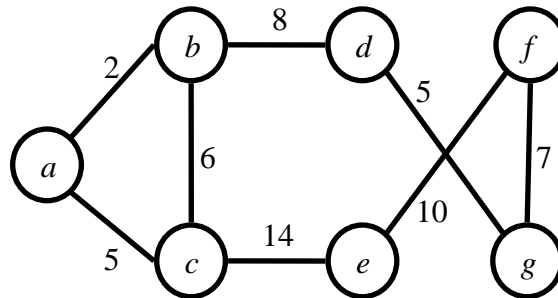
where the PARTITION algorithm takes $\Theta(n)$ time. Suppose the parameter q is selected at $\lfloor n/2 \rfloor$, answer the following questions:

- Model the time complexity of the QUICKSORT algorithm in terms of a recurrence equation. (5%)
 - Solve the recurrence equation to give tight asymptotic notation of the running time. (5%)
 - The above QUICKSORT algorithms use two partitions. If, instead, three partitions are used, is the algorithm more efficient? Explain why by comparing the tight asymptotic notation of the running time. (4%)
8. Suppose we model the following maze by having a vertex for the starting point, the finishing point, dead ends, and all the points in the maze where more than one path can be taken and then connecting the vertices according to the paths in the maze.



- Construct such a graph for the given maze (maintain the relevant locations for the vertices). (3%)
 - Construct the search tree using breath-first search (BFS). (5%)
 - Instead of BFS, construct the search tree using depth-first search (DFS). (5%)
 - Which traversal would be more efficient for solving the maze? Explain why. (3%)
- (Note: Assume the search starts from the starting point and follows the alphabetic order. However, at any point, always search the finishing point first)

9. Given the following graph,
- Find the minimum spanning tree using the Kruskal's algorithm. (5%)
 - Find the minimum spanning tree using the Prim's algorithm (starting from a). (5%)
 - Determine if the minimum spanning tree is unique. (5%)
- (Show your works to get full credits)



10. Let b_n denote the number of different binary trees with n nodes, which can be defined as follows:

$$b_n = \sum_{k=0}^{n-1} b_k b_{n-k}, \text{ for } n \geq 1$$

and $b_0 = 1$.

- Determine the number of different binary trees with 4 nodes. (5%)
 - Design an algorithm using Pseudocodes (or C/C++ program) to compute the number of different binary trees with n as input. (5%)
11. For the following problems, pick the problems that belong to the class NP? (10%)
- All-pairs Shortest Paths;
 - Clique Problem;
 - 3-Coloring of a Graph;
 - Euler's Tour;
 - Hamiltonian Cycle;
 - Longest Common Subsequence;
 - Tower of Hanoi Problem;
 - Fast-Fourier Transform;
 - Traveling-Salesman Problem.