

中原大學 94 學年度碩士班入學考試

3 月 20 日 11:00~12:30 資訊工程系

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：計算機數學

(共 2 頁第 1 頁)

可使用計算機，惟僅限不具可程式及多重記憶者

不可使用計算機

一律作答於答案卷內，不可答題於本試題卷。

1. [10%] True or False. Determine whether each of the following statements is true or false.

(a) Let A , B , and C be sets. Then $A \cap (B - C) = (A \cap B) - (A \cap C)$.

(b) If P and Q are *propositions*, then $(P \wedge Q) \rightarrow P$ is a **tautology**.

(c) If $f: Z \rightarrow Z$ is defined by $f(n) = \left\lceil \frac{n}{2} \right\rceil + 1$, then $f(n)$ is one-to-one.

(d) The relation $R = \{(a, b) \mid a \leq b\}$ on the set of integers is **reflexive**.

(e) $n! \in O(n^n)$.

2. [24%] Fill in the blanks in the following statements.

(a) If the set $A = \{1, \{1\}\}$, then **power set** of A is _____.

(b) Let $F(x)$, $P(x)$, and $M(x, y)$ be the statements “ x is female,” “ x is a parent,” “ x is the mother of y ,” respectively. The statement “if a person is female and is a parent, then this person is someone’s mother” can be expressed as a **logical expression** involving **predicates**, **quantifiers** with a universe of discourse consisting of all people, and **logical connectives** as _____.

(c) A set B is defined recursively by

(i) $0 \in B$

(ii) If $x \in B$, then $x + 5 \in B$.

Use **set builder** notation to describe the set B by stating the properties that uniquely characterize the elements in B . $B = \{x \mid \text{_____}\}$.

(d) Give a **big-O** estimate for $f(n) = \log(n!) + (\log n + 1)^2 + 100\sqrt{n}$, use a simple function of **smallest order**. $f(n) = \text{_____}$.

(e) Let T be a **tree** with 3 vertices of degree 3, 1 vertex of degree 2 and x vertices of degree 1. Then $x = \text{_____}$.

(f) Let F be a **forest** consisting of 10 trees. If F has 123 vertices, then F has _____ edges.

(g) There are _____ 4-digit **decimal** telephone numbers having **one or more repeated digits**.

(h) If there are **five** possible grades, A, B, C, D, and F, the **minimum number** of students required in a class to be sure that **at least six** will receive the same grade is _____.

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3. [8%] Find the solution to the recurrence relation

$$a_n - 12 a_{n-2} - 16 a_{n-3} = -108 n + 261$$

with initial conditions $a_0 = 6$, $a_1 = -3$, and $a_2 = 93$.

4. [8%] How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$, where x_i , $i = 1, 2, 3, 4, 5$ is a *nonnegative integer* such that $0 \leq x_i \leq 10$?

5. [10%] Use Gaussian elimination to solve the following system of linear equations.

$$-x_1 - 2x_2 + 3x_3 - 2x_4 = 0$$

$$2x_3 + x_4 = 0$$

$$x_1 + 2x_2 - 2x_3 = 0$$

6. [15%] (a) Find the standard matrix A for a linear transformation on R^3 ; that is an reflection about the zx -plane, followed by an clockwise rotation of 45° about the positive z -axis. (b) Determine the inverse of A .

7. [15%] (a) Determine the eigen values and spaces of the following matrix B and (b) Solve the system of differential equations.

(a)

$$B = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(b)

$$y'_1 = -y_1 + y_2$$

$$y'_2 = y_1 + y_2$$

$$y'_3 = 2y_3$$

8. [10%] Transform u_1, u_2, u_3 to an orthonormal basis v_1, v_2, v_3 under the Euclidean inner product using the Gram-Schmidt process (beginning from u_1). $u_1 = (1, 0, 0)$, $u_2 = (1, 1, 1)$, $u_3 = (0, 2, -1)$.