

# 中原大學 96 學年度碩士班入學考試

96/03/25 14:00~15:30 土木工程學系生物環境工程組

誠實是我們珍視的美德，  
我們喜愛「拒絕作弊，堅守正直」的你！

科目：環境數學

(共 1 頁第 1 頁)

可使用計算機，惟僅限不具可程式及多重記憶者

不可使用計算機

1.  $[A] = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$ , (a) find the eigenvalues of  $[A]$  (10%), (b) find

the eigenvectors of  $[A]$  (10%) and (c) diagonalize the matrix  $[A]$ . (10%)

2. Let  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  be the position vector of a mass  $m_1$  and let the mass  $m_2$  be located at the origin. If the force of gravitational

attraction is  $\vec{F} = -\frac{G m_1 m_2}{\|\vec{r}\|^3}$

Verify that  $\nabla \times \vec{F} = 0$ , and  $\nabla \cdot \vec{F} = 0$ . (15%)

3. Apply The Laplace transform to find the particular solution of the equation  $y'' - 6y' + 9y = t^2 e^{3t}$ ;  $y(0) = 2$ ,  $y'(0) = 6$  (15%)

4. Find the solution of the following system of equations. (15%)

$$\frac{dx}{dt} - 4x + \frac{d^2 y}{dx^2} = t^2$$
$$\frac{dx}{dt} + x + \frac{dy}{dt} = 0$$

5. Find the direction derivative of  $F(x, y, z) = xy^2 - 3yz + 5xz^5$  at point

$(1, -1, 2)$  in the direction of  $5\vec{i} + 2\vec{j} - 3\vec{k}$ . (10%)

6. Solve the one-dimensional heat equation (15%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 \leq x \leq 1, t > 0$$
$$u(0, t) = 0, u(1, t) = 1, u(x, 0) = x$$