

中原大學 94 學年度碩士班入學考試

3 月 20 日 14:00~15:30

土木工程系結構組/大地組/水環組

科目：工程數學

可使用計算機，惟僅限不具可程式及多重記憶者

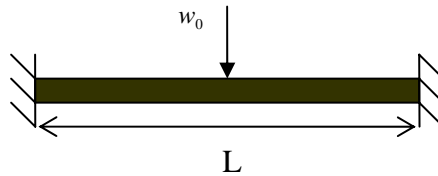
誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

(共 1 頁第 1 頁)

不可使用計算機

1. Solve $y'' - 4y' + 4y = (x+1)e^{2x}$ (10%)

2. A uniform beam of length L carries a concentrated load w_0 at $x = L/2$. The beam is embedded at both ends. Use the **Laplace transform method** to solve the differential equation $EI \frac{d^4 y}{dx^4} = w_0 \delta(x - L/2)$, where $\delta(x - L/2)$ is a Dirac delta function, and $y(0) = 0, y'(0) = 0, y(L) = 0, y'(L) = 0$ (15%)



3. Expand $f(x) = x^3, 0 < x < 1$, in a Fourier Sine series. (10%)

4. (a) Describe the orthogonality of the Sturm-Liouville problems. (5%)

(b) Solve $y''(x) + 9\lambda^2 y(x) = 0$
 $y(0) = 0, y(1) + y'(1) = 0$ (10%)

5. Given $\vec{v}_1 = \vec{e}_1 - \vec{e}_2 + \vec{e}_3, \vec{v}_2 = \vec{e}_1 + \vec{e}_2 + \vec{e}_3, \vec{v}_3 = \vec{e}_1 - \vec{e}_2 - \vec{e}_3$, and

$\vec{e}_1 = \vec{i} + \vec{j} - \vec{k}, \vec{e}_2 = \vec{i} - \vec{j} + 2\vec{k}, \vec{e}_3 = \vec{i} - 2\vec{j} + \vec{k}$, where $\vec{i}, \vec{j}, \vec{k}$ are the unit vectors of the orthogonal curvilinear coordinate system.

(1) judge whether $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a base or not? (10%)

(2) if the answer of (1) is “yes”, find the reciprocal base of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$; if the

answer of (1) is “no”, explain why $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ cannot be a base. (10%)

6. Find the solution of the following integral $I = \int_c \operatorname{csch}^2 z dz$, where $c: |z| = 1$ (15%)

7. Solve $u_t = u_{xx} + F(x)$, with B.C. $u(0, t) = 0, u(L, t) = 0$, and $u(x, 0) = f(x)$, where $F(x)$ and $f(x)$ are given functions. (15%)