

中原大學 97 年度 博士班 入學 考試

97年6月4日 10:30 ~ 12:00 應用數學系 (一般生及在職生)

誠實是我們的珍視的美德,
我們喜愛「拒絕作弊, 堅守正直」的你!

科目: 統計學

不可使用計算機 (共 1 頁 第 1 頁)

(每個問題 10 分)

Let $X_1, X_2, \dots, X_n \stackrel{iid}{\sim} f(x; \theta) = \frac{1}{\theta} I_{(0, \theta)}(x)$, $\theta > 0$, and $W_n = \max\{X_i\}_{i=1}^n$.

(1) Show that $(-1)(\log X_1 + \log X_2)$ follows a Gamma distribution. [10%]

(2) Find the probability density function (*pdf*) of W_n . [10%]

(3) Find the expectation of X_1/W_n ; *i.e.* $E\left(\frac{X_1}{W_n}\right)$. [10%]

(4) Find the conditional expectation $E(X_1 | W_n)$. [10%]

(5) Show that W_n converges to θ in probability. [10%]

(6) Find the maximum likelihood estimator of θ^2 . [10%]

(7) Find the estimator of the form $T(X_1, X_2, \dots, X_n) = bW_n$, $b \in \mathcal{R}^+$,
such that it has the smallest mean squared error. [10%]

(8) Derive a uniformly most powerful size- α test ϕ_1 for testing

$$H_0 : \theta \leq 1 \text{ versus } H_1 : \theta > 1$$

if it exists. [10%]

(9) Determine a size- α the likelihood ratio test ϕ_2 for [10%]

$$H_0 : \theta = 1 \text{ versus } H_1 : \theta \neq 1.$$

(10) Find a 100 $(1-\alpha)$ percent confidence interval for θ . [10%]