中原大學 96 學年度博士班入學考試

96/06/06 08:30~10:00 應用數學系(一般生及在職生)

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：機率

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□可使用計算機，惟僅限不具可程式及多重記憶者 ■不可使用計算機

1. (a) Let $\{Z_k\}$ be a non-negative random variables sequence and $\sum_{k=1}^{\infty} E(Z_k) < \infty$. Prove that $\sum_{k=1}^{\infty} Z_k < \infty$ almost surely.(10%)

   (b) Suppose that $X_1, X_2, \cdots$ are independent random variables, and that for some constant K in $[0, \infty)$, $E(X_k) = 0$, $E(X_k^4) \le K$, $k = 1, 2, 3, \cdots$.
   Let $S_n = X_1 + X_2 + \cdots + X_n$. Prove $\frac{S_n}{n} \to 0$ almost surely. (15%)

2. Suppose that $X \in \mathcal{L}^1(\Omega, \mathcal{F}, \mathcal{P})$. Prove given $\varepsilon > 0$, there exists a $\delta > 0$ such that for $F \in \mathcal{F}$, $\mathcal{P}(F) < \delta$ implies that $\int_F |X| d\mathcal{P} < \varepsilon$. (15%)

3. Let $\{X_n\}$ be a sequence in $\mathcal{L}^1(\Omega, \mathcal{F}, \mathcal{P})$, and let $X \in \mathcal{L}^1(\Omega, \mathcal{F}, \mathcal{P})$.
   Prove $X_n \to X$ in $\mathcal{L}^1(\Omega, \mathcal{F}, \mathcal{P}) \Rightarrow$ the sequence $\{X_n\}$ is Uniform Integrability and $X_n \to X$ in probability. (15%)

4. Let $\{X_n\}$ be a sequence of random variables, and let X be a random variable.
   Prove $X_n \to X$ almost surely $\Rightarrow X_n \to X$ in probability. (15%)

5. Let $\{F_n\}$ be a sequence of distribution functions on $\mathbf{R}$, and let F be a distribution function on $\mathbf{R}$.
   Prove F_n converges weakly to F if and only if $\lim_{n\to\infty} F_n(x) = F(x)$ for every point of continuity x of F. (15%)

6. Let $1 \le p \le r < \infty$ and $Y \in \mathcal{L}^r(\Omega, \mathcal{F}, \mathcal{P})$. Prove $Y \in \mathcal{L}^p(\Omega, \mathcal{F}, \mathcal{P})$ and $\{E|Y|^p\}^{\frac{1}{p}} \le \{E|Y|^r\}^{\frac{1}{r}}$.(15%)